PUMP DESIGN

J.A. de Jongh & R.P.P. Rijs, editors

ARRAKIS
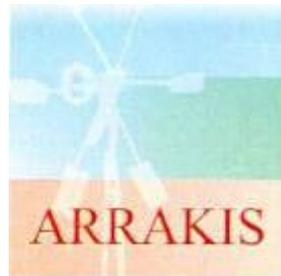
Latest update: March 2004

CONTENTS

1 Classification of Water Pumps

Water pumps can be divided into three types: displacement, impulse and other types.

Positive displacement pumps can be of reciprocating and rotary type. In either case liquid is displaced from the low pressure suction side to the high pressure discharge side (the term positive refers to the direction of flow displacement related to the pressure gradient). The geometry of the pump is changed periodically and determines the flow in both supply and delivery system. In a positive displacement pump there is no direct communication between the suction and discharge circuit. As a rule, a positive displacement pump is self-priming. A general classification of pump is given in the Figure 1.
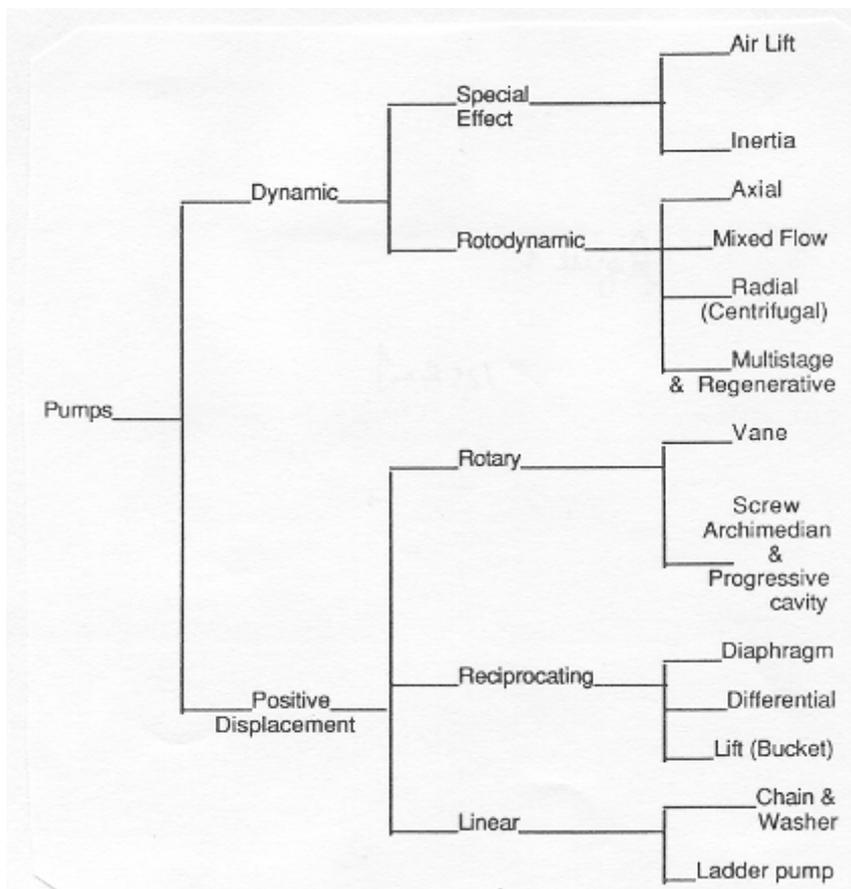
Figure 1   Classification of pumps according to Burton.

1.1 Piston Pumps

A reciprocating piston pump basically consists of a piston, two valves and a suction and a delivery pipe. Sometimes airchambers are applied to smooth the flow and reduce shock forces. In the traditional piston pump the upper valve is usually situated in the piston and known as the piston valve; the lower valve is called the foot valve. If the upper valve is not integrated in the piston, the pump is usually called a plunger pump (Figure 2).
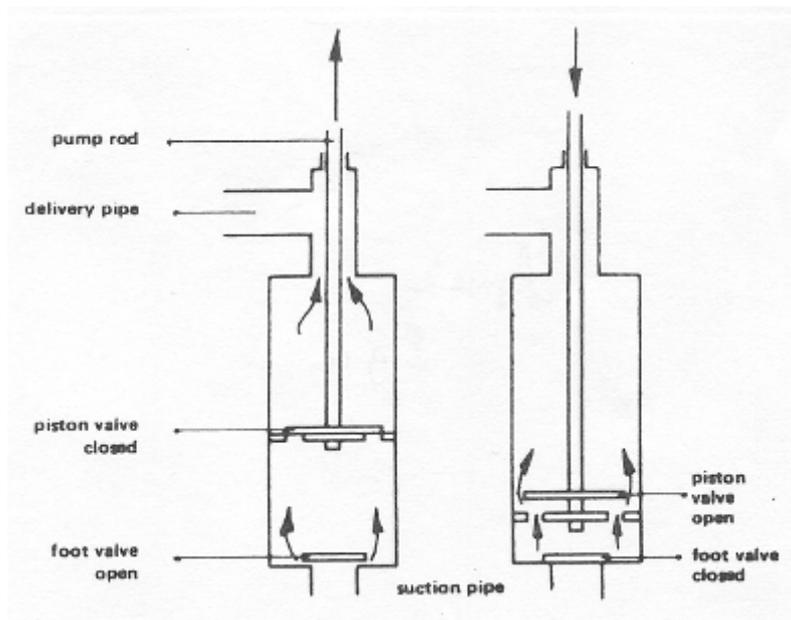
Figure 2   Working principle of a single-acting reciprocating piston pump.

The operation principle of the reciprocating piston pump is simple: in the downstroke, the piston valve is opened and the water passes through it while the piston moves downward. During the downstroke, the foot valve remains closed. In the upstroke, the upper valve is closed and water being lifted above the piston. Meanwhile, the volume under the piston is being repleted with water passing through the opened foot valve. The result is a pulsating water flow (Figure 3).
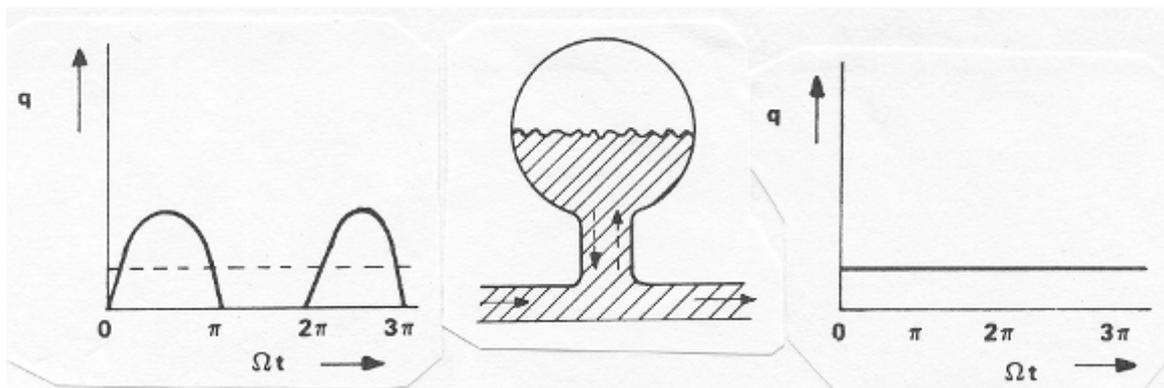
Figure 3   Pulsating output flow of a single-acting piston pump (a), and rectified flow after an airchamber (b).

Apart from this so-called single-acting pump, also double-acting pumps exist, with two pistons moving in opposite direction giving a more constant output delivery. A double-acting operation is also obtained by the so-called differential pump as shown in Figure 4. Since the rod cross sectional area A_{pr} is half that of the rising main A_{rm}, equal volumes of water are discharged on both the up and the downstroke. This type of pump was applied in the Colombian Gaviotas pump, of which several thousands have been built. The disadvantages of these pumps is that there is compressive force on the pump rod during the downstroke and the higher complexity. This makes them not attractive for deepwell pumping.

Piston pumps for water supply can be mounted above the water level in the source or below it (submerged). A surface-mounted piston pump works by suction, as the water is lifted to the pump level by the under-pressure under the piston. This principle only works for pumping heads upto 6-7 metres. For greater pumping heads, the pump (piston, cylinder and valves) must be installed at the bottom of the well and actually lifts the water.
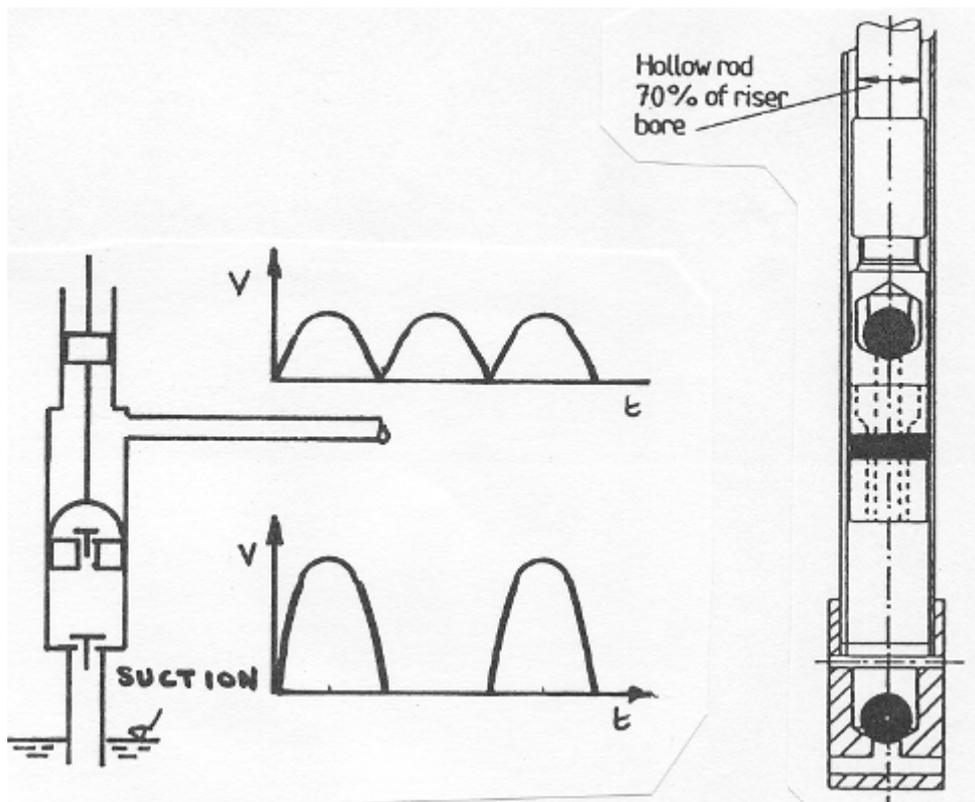
Figure 4   Working principle of a differential (double-acting) piston pump (a) and geometry of the Colombian Gaviotas pump (b).

As water is lifted only during the upward stroke, the torque (and power) requirement of a single-acting piston pump varies cyclically. During the upward stroke the piston is subject to the full water pressure. During the downward stroke the piston valve opens and the torque is virtually zero. For this reason, piston pumps need a much higher torque to start than the average torque required to keep them running. The average torque required to operate a piston pump is almost independent of the pump speed, as it only depends on the gravity force of the (constant) volume of the water being lifted during the upward stroke.

This behaviour influences adversely the number of hours a windpump supplies water. Various techniques are used to improve the starting behaviour of a single-acting piston pump, such as:

- counterbalancing the pump rod weight
- drilling a leak-hole in the piston
- use of hydraulic seals instead of cups
- applying a spring in pre-tension
- use of a floating valve
- variable stroke mechanism

1.2 Centrifugal Pumps

Centrifugal (or rotodynamic) pumps are based on the principle of imparting kinetic energy to the water. In these pumps water enters axially and is discharged by the rotor into a discharge pipe. They have an impeller which rotates in a casing of a special shape. The impeller vanes accelerate the water, which is thrown out by the centrifugal force. The shape of the casing is designed to effectively build up a high pressure at the pump outlet. It is this pressure level that lifts the water against the pumping head. In Figure 5, a single stage of a centrifugal pump is shown. This type of pumps are typically driven by an electric motor or combustion engine and installed above ground level.
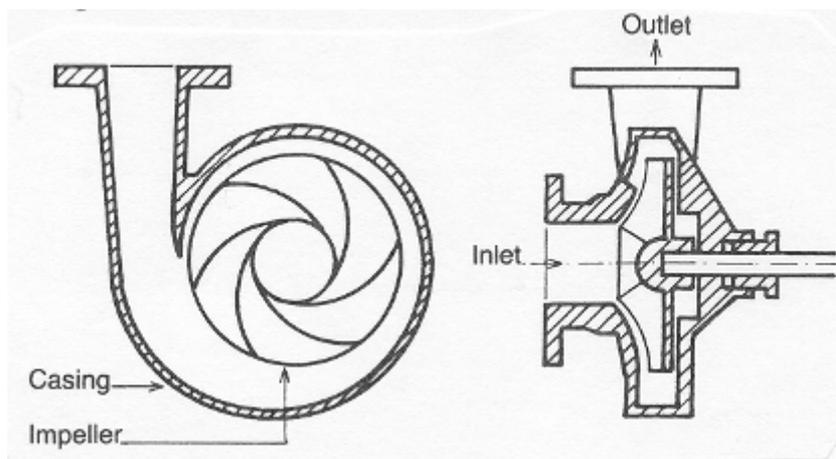
Figure 5   Impeller and casing of a centrifugal pump stage.

Each impeller together with its casing is called a stage. If more pressure is needed than can be created by a single stage, several stages can be mounted in series on a common shaft to form a multi-stage pump. As the water passes through the successive stages, pressure is built up until the required pressure head is obtained. Multi-stage centrifugal pumps with a small diameter and integrated electric motor can be lowered into a borehole for deepwell pumping. Such pumps are used in conjunction with photovoltaic and wind-electric systems.

The efficiency of centrifugal pumps varies strongly with the output rate for a given head. Centrifugal pumps are designed for optimal operation in their design point, i.e. at one determined pressure head and specific flow rate. Away from this design point, their efficiency drops rapidly. In windpumps, piston pumps are much more common than centrifugal pumps since their efficiency is less sensitive to fluctuations in head and speed.
Centrifugal pumps are mostly found for low-head applications where piston pumps perform poorly. Of the very few windpumps equipped with a centrifugal pump, the Dutch Bosman is widely used in The Netherlands for drainage purposes. Experimental designs have been developed for medium pumping heads, but have not been commercialised.

2 The Single-acting Piston Pump

2.1 Torque and Flow

A single-acting piston pump without airchamber, directly coupled to a wind rotor, is depicted in Figure 6. The rotor shaft drives the pump through a crank mechanism; therefore the rotational speed of the pump is equal to that of the rotor and the pump stroke dictated by the eccentricity of the crank mechanism.
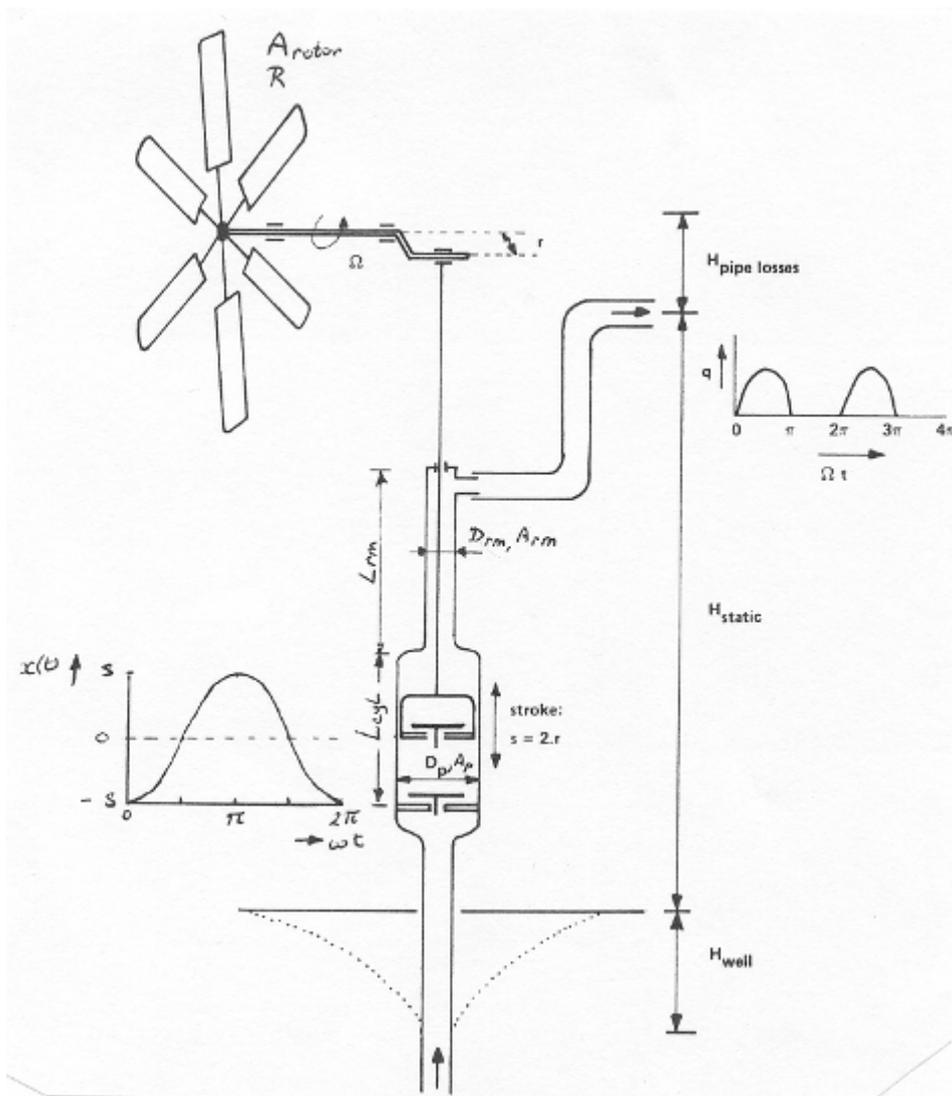
Figure 6   Single-acting piston pump directly coupled to a wind rotor. Indicated are the piston displacement and rising main and pump dimensions.

The force on the piston during the upstroke is equal to the weight of the water column above it, or the product of piston area A_p and the static head H:

$$F_p = \rho_w \, g \, H \, A_p$$

(1)

More exactly, this expression only holds for a non-moving (static) piston. In a piston pump however, the piston is raised and lowered cyclically in the pump cylinder and has a velocity and acceleration that vary cyclically. The non-zero velocity gives rise to friction forces, while the acceleration causes additional forces to accelerate the water column on the piston. For the time being, we consider a quasi-stationary, slowly turning piston pump and neglect friction and acceleration forces.

The displacement of the piston $x_p(t)$ with time is, taking the neutral position of the piston at the bottom dead centre as shown in Figure 6:

$$x_p(t) = -\tfrac{1}{2}s - \tfrac{1}{2} s \cos(\omega t)$$

(2)

Note that ω is the pump frequency in radians per second; the stroke frequency n (in strokes or cycles per second) follows from:

$$n = \omega/(2\pi)$$

(3)

The stroke volume of the pump is equal to the product of stroke and piston area $s A_p$. This is also the volume of water that is lifted during each pump cycle. Hence the average flow q_{av} is equal to the product of stroke volume and pump frequency according:

$$q_{av} = \omega/(2\pi) \, s \, A_p$$

(4)

As the piston displacement is directly determined by the crank length of the eccentric at the rotor shaft, the pump torque during the upstroke is equal to:

$$Q_{pump}(t) = \rho_w \, g \, H \, A_p \, \tfrac{1}{2} \, s \, \sin(\omega t)$$

(5)

Hence, the pump torque reaches its maximum at the middle position of the piston, and is zero at the upper and lower dead centre.

In the downstroke, the pump valve is opened and the weight of the water column rests on the foot valve. Then the force on the piston and the torque are zero in this idealized situation.

The maximum value of Q_{pump} is the torque required for the pump to start running and is equal to the weight of the water column on the piston times the crack length:

$$Q_{start} = \tfrac{1}{2} \, s \, \rho_w \, g \, H \, A_p$$

(6)

By integrating $Q_{pump}(t)$ over a full cycle (one upstroke and one downstroke) we obtain the average pump torque Q_{av}:

$$Q_{av} = 1/(2\pi) \; \rho_w \; g \; H \; s \; A_p$$

(7)

The average pump torque Q_{av} for a well-designed piston pump is fairly independent of the pumping speed. By comparing *(6)* and *(7)* we find that: a piston pump requires a torque to start it equal to π times the torque required to keep it running:

$$Q_{start} = \pi \, Q_{av}$$

(8)

This result largely determines the starting behaviour of a windpump and reduces the number of running hours. In practice the staring torque may even be higher due to the weight of the piston and the pump rod and friction between the cup sealings and the cylinder wall. This puts the need for starting devices into evidence.

2.2 Volumetric and Mechanical Efficiency

In a practical situation, the average output as found in *(4)* may be different due to leakage, delayed valve response and inertia effects. We define the volumetric pump efficiency η_{vol} as the ratio between the effective average output flow and the above derived one, hence:

$$q_{av, \, eff} = \eta_{vol} \; \omega/(2\pi) \; s \; A_p$$

(9)

At low speeds of operation the volumetric efficiency is normally less than 100% due to water leakage over the piston and the valves and to delayed closing of the valves. At high speeds the volumetric efficiency may be above 100% due to the inertia of the water flow: at the end of the upstroke the water has gained so much momentum that it continues moving upward (through the valves which remain opened) during the downward stroke. Volumetric efficiencies under design conditions are typically 80% to 90%.

The average hydraulic power P_{hyd} is defined as the nett power represented by the lifted water:

$$P_{hyd} = q_{av, \, eff} \; \rho_w \; g \; H$$

(10)

The mechanical efficiency η_{mech} is found from the ratio between the hydraulic power P_{hyd} and the (average) mechanical power P_{mech} to drive the pump. This efficiency is less

than 100% because of mechanical losses due to friction between piston and cylinder wall and hydraulic losses due to flow drag, mainly in the valves. The mechanical efficiency is given by:

$$\eta_{mech} = P_{hyd}/P_{mech}.$$

Now, by combining the expression for the hydraulic output *(10)* and the expression for the shaft power delivered by the rotor:

$$P_{mech} = \tfrac{1}{2}\, \rho\, C_p\, A_{rotor}\, V^3$$

(11)

we obtain for the mechanical efficiency of the windpump:

$$\eta_{mech} = (q_{av,\ eff}\, \rho_w\, g\, H) / (\tfrac{1}{2}\, \rho\, C_p\, A_{rotor}\, V^3)$$

(12)

Mechanical efficiencies of 80% to 90% are attained in high head pumps, in which the pressure losses due to flow drag are small compared to the static pressure. For low heads, the efficiency drops to 60% to 70% since the losses are relatively independent from the head (but the hydraulic output decreases proportionally to the head).

2.3 Non-stationary Behaviour and Valve Response

The relations derived above only hold for a slow movement of the piston. At higher pump speeds (above one stroke per second), acceleration effects become notable in the water flow, causing a number of complications due to cavitation, pump rod buckling, inertia flow, shock forces in the pump rod, and wave phenomena. In this section, cavitation and buckling are shortly dealt with, while pump rod forces are more extensively dealt with in section 4.

From the expression for the piston displacement *(2)* we obtain the piston speed v_p by differentiating:

$$v_p(t) = \tfrac{1}{2}\, s\omega\, \sin(\omega t)$$

and the acceleration:

$$a_p(t) = \tfrac{1}{2}\, s\omega^2\, \cos(\omega t)$$

The maximum value or amplitude of the piston acceleration a_p is equal to

$$a_p = \tfrac{1}{2} \, s\omega^2$$

This piston acceleration a_p competes with the gravity and it proves convenient to introduce an acceleration coefficient c_a according:

$$c_a = \tfrac{1}{2} \, s\omega^2/g$$

(13)

This ratio should be kept well below 1 in order to avoid dynamic complications.

2.3.1 <u>Pump Rod Buckling</u>

If the ratio c_a becomes higher than 1, the pump rod is forced to accelerate downward during the downward stroke faster than the gravity would do. A compressive force is exerted on the pump rod and it may buckle. Since cup friction and flow drag must also be overcome during the downstroke, buckling may start at an acceleration ratio lower than 1. As a safe rule, this factor should not exceed 0.5, hence:

$$s\omega^2 \leq g$$

It is seen that the piston acceleration can be reduced by lowering the pump speed and increasing the stroke. Especially small wind rotors have high rotational speeds and most commercial types are therefore equipped with a transmission.

2.3.2 <u>Cavitation</u>

In suction pumps the acceleration of the water column below the piston decreases linearly with the head. In fact, if the pumping head becomes equal to the atmospheric pressure, no acceleration will occur at all. The situation for a suction pump with and without an airchamber is depicted in Figure 7.
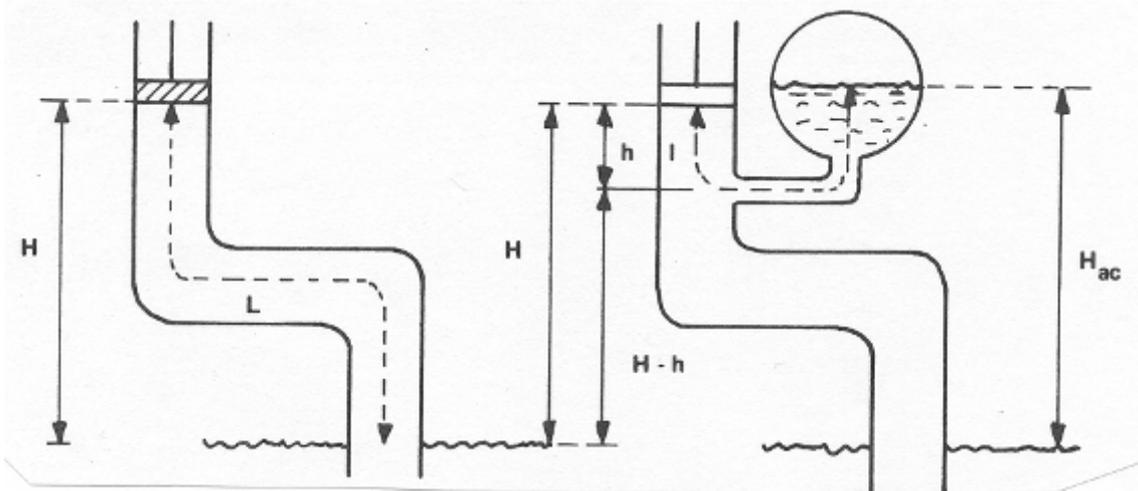
Figure 7   Influence of the suction head H and the length of the suction pipe L on the occurrence of cavitation without (a) and with an airchamber (b).

The pressure difference over the water column depends on the atmospheric pressure and the suction head H. The driving force is given by the pressure difference and the piston area according (assuming an atmospheric pressure of 10 meter water column):

$$F = \rho_w \, (10 - H) \, A_p$$

The mass M of the water column to be accelerated depends on the length of the suction pipe L, which may be larger than the head H as illustrated in the figure:

$$M = \rho_w \, L \, A_p$$

We find for the acceleration of the water column:

$$a_w = (10 - H)/(L \, g)$$

(14)

In order to follow the piston movement, this acceleration a_w must at any time be larger than the piston acceleration a_p, hence:

$$(10 - H)/L \, g \geq \tfrac{1}{2} \, s \omega^2$$

This shows that in case of a 5 m head and 10 m suction pipe, the maximum piston acceleration cannot be larger than 0.5 g. The best way to avoid cavitation is not to use high suction heads or, if this is not possible, to reduce the length L of the water column concerned. This can be done by introducing an airchamber near the pump in the suction

pipe; in this case for *L* the distance between the piston and the airchamber should be taken.

2.3.3 *Inertia Flow*

When the deceleration of the piston during the last part of the upstroke is larger than the gravity acceleration, the water column will open the piston valve and continue to move upward slowed down by the gravity only. This means that the pump will displace more water than would be expected from the stroke volume and the volumetric efficiency may achieve values higher than 100%.

3 Coupling of a Pump to a Wind Rotor

A piston pump coupled to a windmill together form a system in which both components influence each other. The behaviour of this system is complicated even more by external parameters, such as the pumping head and the windmill safety system.
In general, by choosing a large pump high volumes of water are obtained but the windmill will not run most of the time. In other words, the windpump has a high output but a low availability. A small pump coupled to the same windmill will have a higher availability but a lower output. In a practical situation, one tries to find the most appropriate compromise between output and availability. This process of tuning the system components to each other and to the prevailing wind regime, is called matching.

In a stationary situation, matching of a windpump system is a rather straightforward procedure and formulas can be derived to determine the optimal combination. The windspeed itself is a stochastic variable, but with the aid of wind regime (Weibull) distributions, a mathematical expression can be obtained for the longterm water output and availability. For the practice of windpumping, a first complication is the presence and duration of lulls, i.e. consecutive days with low winds and no water. The occurrance of periods with low windspeeds is not covered by the Weibull description, yet they determine which size of storage tank is needed.
However, the operation of a windpump in practice is non-stationary. As windspeed is fluctuating, the wind rotor will accelerate and decelerate constantly. Due to the inertia of the system, the wind rotor will follow the windspeed pattern, but with a certain delay. If the windspeed falls below a certain value, the windmill will gradually come to a standstill. If the windspeed increases, at a certain point the windmill will be strong enough and start turning again. This behaviour is much more difficult to describe analytically. Prolonged measurements, or detailed time-series computer simulations are needed to analyze the system behaviour and calculate the long term output and availability. As a wind regime may vary from one site to another, output calculations are related in principle valid only for a specific site.

In practice the approach is rather straightforward, for the sake of simplicity and because

exact wind data are almost never available. However, computere simulations and long term observations may provide us with practical guidelines for component matching. The concept of the design wind speed is introduced and will serve as a parameter to be compared with the local wind speed. The design wind speed is dealt with in the following section.

3.1Design Point Matching

In a windpump, the mechanical power demanded by the piston pump at any point of operation equals the shaft power delivered by the wind rotor:

$$P_{mech} = P_{rotor}$$

As found in *(12)*, the effective hydraulic output (flow of water lifted) is equal to the mechanical power times the mechanical efficiency of the transmission and pump. For convenience we drop the subscript for the efficiency and write:

$$P_{hyd} = \eta\, P_{mech}$$

By using the expressions for the hydraulic power *(10)* and applying the expression for the rotor power according aerodynamic theory:

$$P_{rotor} = (C_p\ \eta)\ \tfrac{1}{2}\ \rho\ V^3\ \pi R^2$$

(15)

we obtain for the momentaneous power balance:

$$(C_p\ \eta)\ \tfrac{1}{2}\ \rho\ V^3\ \pi R^2 = q_{av,\ eff}\ \rho_w\ g\ H$$

(16)

Now, the design wind speed V_d of the windpump is defined as the wind speed for which the system operates at maximum efficiency. Since the mechanical efficiency of a piston pump is more or less constant, this working point coincides with the point at which the rotor operates at its design tip speed ratio. At the design point, we prescribe a design output flow q_d and find:

$$(C_p\ \eta)_{max}\ \tfrac{1}{2}\ \rho\ V_d^3\ \pi R^2 = q_d\ \rho_w\ g\ H$$

(17)

The flow $q_{av,\ eff}$ depends on the stroke volume, the pump speed and the volumetric efficiency according *(9)*. At the design point, we choose a design pump speed ω_d in congruence with the design tip speed ratio λ_d and obtain:

$$q_d = \eta_{vol} \, s \, A_p \, \omega_d \, /(2\pi)$$

$$= \eta_{vol} \, s \, A_p \, \lambda_d \, V_d /(2\pi R)$$

(18)

By substituting *(18)* into *(17)* and rearranging we find an expression for the design wind speed of the rotor-pump combination:

$$V_d = \sqrt{(\eta_{vol} \, s \, A_p \, \lambda_d \, \rho_w \, g \, H) \, / \, ((C_p \, \eta)_{max} \, \rho \, \pi^2 \, R^3)}$$

(19)

For a given wind rotor and appropriate choice for V_d, the corresponding pump volume is calculated from:

$$s \, A_p = ((C_p \, \eta)_{max} \, \rho \, \pi^2 \, R^3) \, /(\eta_{vol} \, \lambda_d \, \rho_w \, g) \, (V_d^2 /H)$$

(20)

It should be stressed that the expression for the design wind speed does not give any information about the long term water production of a windpump. It is a direct relation between the chosen design water output q_d (in m³/day) and a chosen design windspeed V_d. At this wind speed and output, the conversion efficiency of the windpump is maximal -that is why this working point is called the design point.
Due to the fluctuating character of the wind, a windpump will be operating at off-design conditions most of the time. The long term water production and the number of running hours is found by combining the complete power curve (output versus wind speed) of the windpump and the wind regime at the site. However, the choice of V_d relative to the prevailing average wind speed V_{av} provides a simple means to optimize the performance of a windpump.

3.2 Long Term Output and Output Availability

In section 3.1 the relation between the area of the wind rotor and the pump volume was derived for design point operation. However, most of the time a windpump is running at off-design conditions in response to the fluctuations in wind speed. Therefore, the average output depends on the windpump characteristics as well as on the local wind conditions.

As a simple rule of thumb, the long term hydraulic output of a windpump is given by the relation:

$$P_{av, \, hyd} = 0.1 \, V_{av}^3 \, \pi R^2$$

(21)

This formula may be used as a guideline for a properly matched windpump. The other way around, it is also useful as a first estimate to determine the windpump diameter (though the exact diameter should be determined on basis of the critical month).

As the hydraulic power is equal to (both short term and long term):

$$P_{hyd} = q_{av,\,eff}\,\rho_w\,g\,H$$

(10)

by combining we find for the average output flow

$$q_{av,\,eff} = (0.1\,V_{av}{}^3\,\pi R^2)\,/\,(\rho_w\,g\,H)$$

(22)

and:

$$= 1/98,000\,(V_{av}{}^3\,\pi R^2)\,/\,H \quad [m^3/s]$$

For the output flow $q_{av,\,eff}$ in litres per second we obtain approximately:

$$q_{av,\,eff} \approx 1/100\,(V_{av}{}^3\,\pi R^2)\,/\,H\ [l/s]$$

so the expected output for a properly matched windpump in litres per second and per unit rotor area, becomes:

$$q_{av,\,eff}\,/A_{rotor} \approx 1/100\,(V_{av}{}^3\,/\,H) \qquad [l/s/m^2]$$

(23)

For the output flow $q_{av,\,eff}$ in cubic metres per day we obtain:

$$q_{av,\,eff} \approx (V_{av}{}^3\,\pi R^2)\,/\,H \qquad\qquad [m^3/day]$$

so the expected long term output in cubic metre per day per unit rotor area we find, underestimating the output by about 10%:

$$q_{av,\,eff}\,/A_{rotor} \approx V_{av}{}^3\,/\,H \qquad\qquad [m^3/day/m^2]$$

(24)

The output availability of a windpump refers to the probability that a windpump is actually pumping water. More precisely, this is the technical availability of the windpump alone, i.e. without considering if this water is pumped at a moment that there is a need for it. Commonly a minimum amount of water output is chosen as a reference (threshold) value; an output smaller than this value being considered as not useful

anymore. Then the output availability can be calculated from the frequency distribution of the wind and the output curve of the windpump.

4 Pump Rod Forces

The forces in the transmission rotor-pump depend on the pump speed. The safety vane of the windpump is designed to keep the rotor and pump speed below a certain value above the rated wind speed V_{rated}. If there is no transmission gearbox, the rotational speed of the pump is equal to that of the wind rotor. The whole transmission (head and pump rod), the pump itself as well as the tower structure must be capable to withstand these forces.

In practice, the maximum rotor (and pump) speed will not be limited to the steady-state value at V_{rated}, but will exhibit overshoots due to the inertia of the rotor itself and the head of the windpump. The graphs below show some measurements, which compare the maximum occurring pump speeds with the steady-state values imposed by the safety mechanism. As we see, the actual occurring pump speeds may be twice as high, and these maximum value should be taken to calculate the strength of the transmission system.
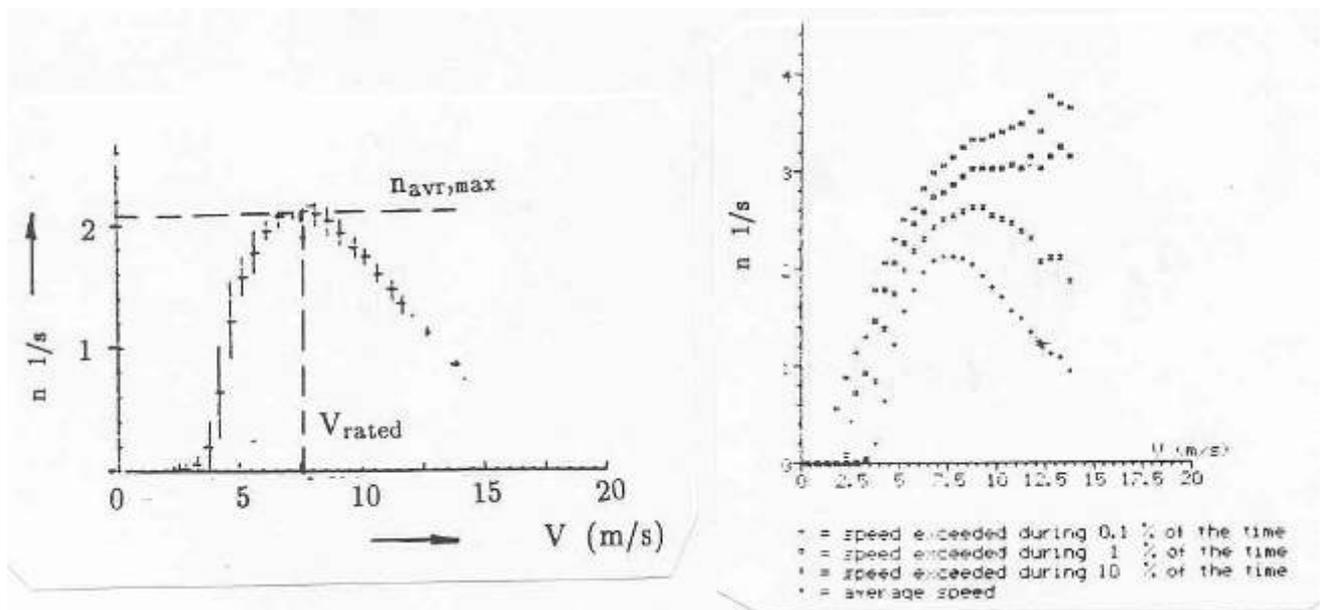

Figure 8   Frequency distribution of rotational speeds of a CWD 2740 windpump versus the wind speed averaged over 10 minutes. Graph (a) gives the average rotational speed, graph (b) the short term fluctuations around the average value. It is seen that the safety systems is not perfect (the average speed is not constant above V_{rated}.

4.1 Examination of Pump Rod Forces

The total pump rod force can be thought of as the result of the following components.

Static forces:

- the weight of the water column lifted by the piston
- the weigth of the pump rod itself

Dynamic forces:

- force to accelerate / decelerate pump rod
- force to accelerate water column

Friction:

- cup friction at the cylinder wall
- flow drag

Transient forces

- due to finite stiffness of pump rod and transmission
- due to delayed valve closure

Figure 9 gives an indication of the variation of the pump rod force during one pump cycle. Due to the cyclical varations, the pump rod force is a fatigue load. It is clear that the maximum force occurs shortly after passing the lower dead centre when the piston starts moving upwards again. At that point the valve hast just closed and the water column on top of the piston is accelerated. In fact, the pump rod force is gradually built up as the pump rod acts as a spring.
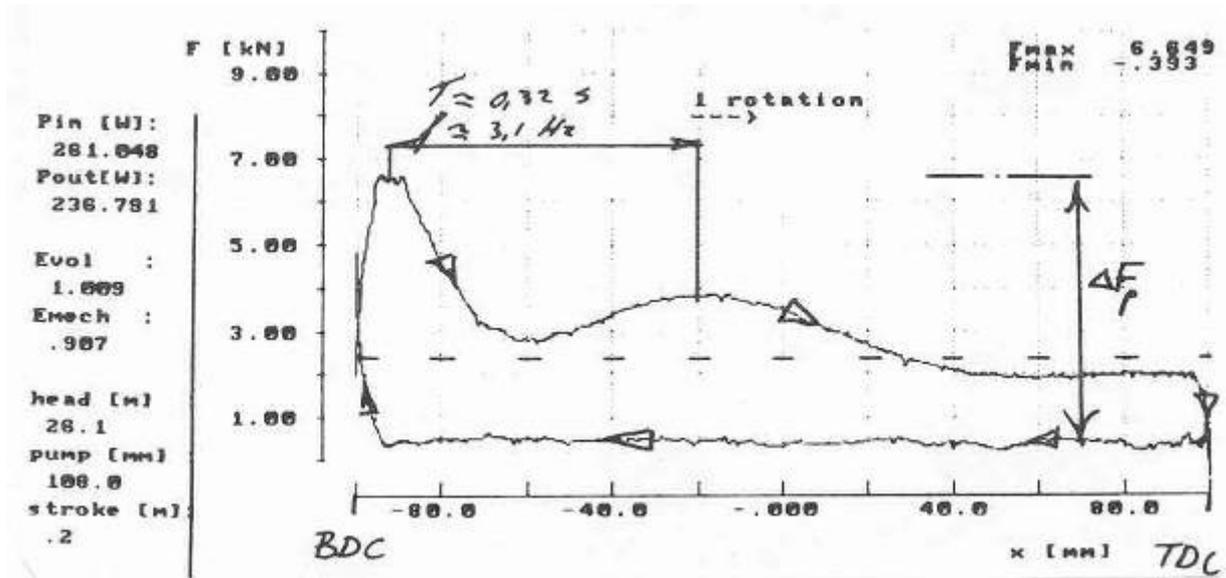
Figure 9   Variation in pump rod loading during one pump cycle.

Of the eight force components only the static ones are, by definition, time-independent. In the following, we only consider the static and dynamic forces to determine an expression for the maximum load on the pump rod. The friction forces are negligible in a well-designed system. The transient forces are important but more difficult to capture in a simple model. We will just assume that they are proportional to the calculated components, so that they can be accounted for by introducing an overshoot factor *k*.

The acceleration forces increase with increasing pump speed. The pump rod itself accelerates and decelerates in symphony with the pump piston. During the deceleration phase in the upstroke and acceleration phase in the downstroke, buckling of the pump rod may occur. For the water column, only the acceleration phase in the upstroke contributes to the pump rod load.

The static force $F_{st, w}$ due to the water column is equal to the weight as already found in *(1)*:

$$F_{st, w} = \rho_w \, g \, H \, A_p$$

(25)

The amplitude of the acceleration force $F_{acc, w}$ of the water column is given by (see Figure 9):

$$F_{acc, w} = \tfrac{1}{2} \, s \, \omega^2 \, \rho_w \, L_{rm} \, A_p^{\,2}/A_{rm}$$

(26)

The sum of these components gives the maximum total pump rod force F_{pr}. Now, by introducing an overshoot factor to account for transient effects, the total pump force will be equal to:

$$F_{pr} = k \, (F_{st,\,w} + F_{acc,\,w})$$

(27)

It is convenient to relate the importance of these components to the static force of the water column $F_{st,\,w}$. Then *(27)* reduces to:

$$F_{pr} = k \, F_{st,\,w} \, (1 + F_{acc,\,w}/F_{st,\,w})$$

(28)

The ratio between the acceleration and the static force due to the water column is equal to:

$$F_{acc,\,w}/F_{st,\,w} = (\tfrac{1}{2} \, s \, \omega^2 \, \rho_w \, L_{rm} \, A_p{}^2/A_{rm})/(\rho_w \, g \, H \, A_p)$$

$$= 1/(2g) \, (A_p/A_{rm}) \, (L_{rm}/H) \, s \, \omega^2$$

By taking the length of the rising main L_{rm} approximately equal to the pumping head H, this expression reduces to:

$$= \tfrac{1}{2} s \, \omega^2/g \, (A_p/A_{rm})$$

(29)

= (acceleration pump rod) / (gravitational acceleration) x (piston area) / (rising main area)

Note that in this expression, the factor $\tfrac{1}{2} s\omega^2/g$ is again the acceleration coefficient c_a as found in *(13)*. If c_a is well below 1, the pump rod force is mainly determined by the static weight of the water column. With increasing c_a, the acceleration component becomes increasingly important.

By substituting this result into *(27)* we finally obtain:

$$F_{pr} = k \, F_{st,\,w} \, [\, 1 + \tfrac{1}{2} \, s \, \omega^2/g \, (A_p/A_{rm}) \,]$$

(30)

or, by using the acceleration coefficient c_a:

$$F_{pr} = k \, F_{st,\,w} \, [\, 1 + c_a \, (A_p/A_{rm}) \,]$$

(31)

The overshoot factor *k* depends on transient effects resulting from the elasticity of the system and the valve response. Its value may be taken equal to 1.5 to 2. The expression in *(30)* gives a rather simple relation to estimate the maximum pump rod force.

4.2 Pump Rod Strength

Once the maximum pump rod force has been found, the minimum pump rod diameter can be calculated. As the pump rod is generally built up of a number of sections that are welded or bolted together, the stresses in the weakest spot may not exceed the maximum allowable values. If the pump rod couplings are welded to the rod, the welds are almost certainly the weakest spot, as depicted in Figure 10. Failure may be expected to occur next to the weld, so this is the critical diameter.
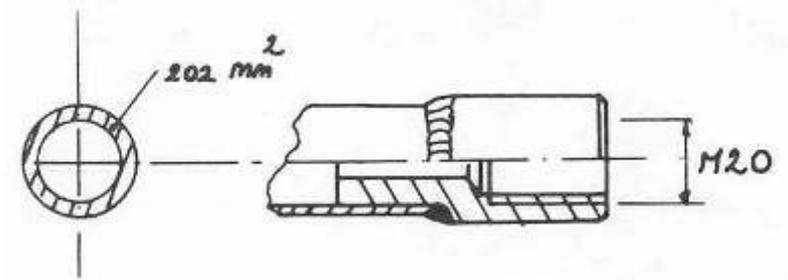
Figure 10   Critical rod coupling geometry.

For a given pump rod material, treatment and fatigue conditions, one may find the maximum admissable stress value from tables. Then the minimum required cross-section can be calculated from:

$$A_{min} = F_{total}/\sigma_{max}$$

(32)

The other way around, the same formula can be used to check whether the maximum occurring stresses in the cross-section are still admissable, if the pump rod force is changed by changing the operating conditions of the windpump.

5 Choosing the Pump Size

As seen in the previous chapters, the size of the pump -for a given, pumping head, wind rotor and wind regime- determines the design wind speed and by consequence:

- the average hydraulic output
- the output availability of water
- the fatigue load on the construction (especially the pump rod)

If a large pump volume is chosen, the design wind speed is increased. This implies a higher water output at design conditions and generally a higher long term output. However, the mechanical loads on the system are increased as well. If the design wind speed is higher, the starting and stopping wind speed are also higher. The windpump will start later and produces much less water during the hours with low wind speeds. The output availability reduces significantly with increasing design wind speed. The most appropriate pump volume is therefore always a compromise between high output and acceptable availability.

Once the pump volume is chosen, a combination of pump diameter and stroke length must be found. The choice of the pump diameter is usually limited, especially if one plans to use a commercial piston pump. Also the size of a selfmade pump depends on the available material for the cylinder. In addition, for deepwell pumping the maximum size is limited by the diameter of the borehole.
Within these limits, one is free to choose a combination of pump stroke and diameter. This choice is of influence on the forces in the pump rod (and as such, on the tower as well). Special attention should be paid to the pump rod loading if the stroke of an installed windpump is increased later on to have more output. Then the loads are increased as well and parts may soon appear to fail.

5.1 Matching the Design Wind Speed

The design wind speed as described in section 3.1 is the wind speed at which the windpump has its best efficiency. For a given wind rotor, average wind speed and pumping head, the design wind speed is chosen to obtain an acceptable compromise between technical output and availability. Then the pump volume is found according *(20)*:

$$s\,A_p = ((C_p\,\eta)_{max}\,\rho\,\pi^2\,R^3)\,/(\eta_{vol}\,\lambda_d\,\rho_w\,g)\,(V_d^2/H)$$

(20)
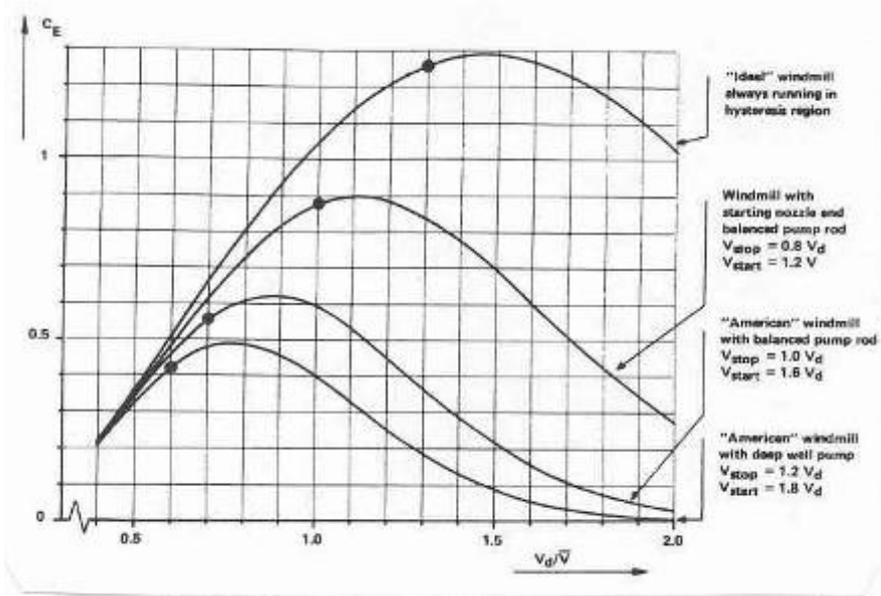
Figure 11   Dimensionless (technical) output of a windpump system with single-acting piston pump in relation to the matching ratio *Vd/Vav* for Weibull factor k = 2.

The calculation of the technical output and availability requires information about the local wind regime and assumes a known behaviour of the safety system. The outcomes are different for every wind regime (Weibull shape factor). A good value to start with is to take the design speed equal to the annual average wind speed. Then, if the availability is unsatisfactory, one may reduce the design speed by shortening the stroke length. If the output is too low, it may be increased by taking a higher design speed by using a larger stroke (and the availability will be reduced significantly).

Figure 11 and Figure 12 give values for the output and availability in relation to V_d/V_{av}. It is seen that the acceptable choices are between:

$$0.8 < V_d/V_{av} < 1.2$$

(33)
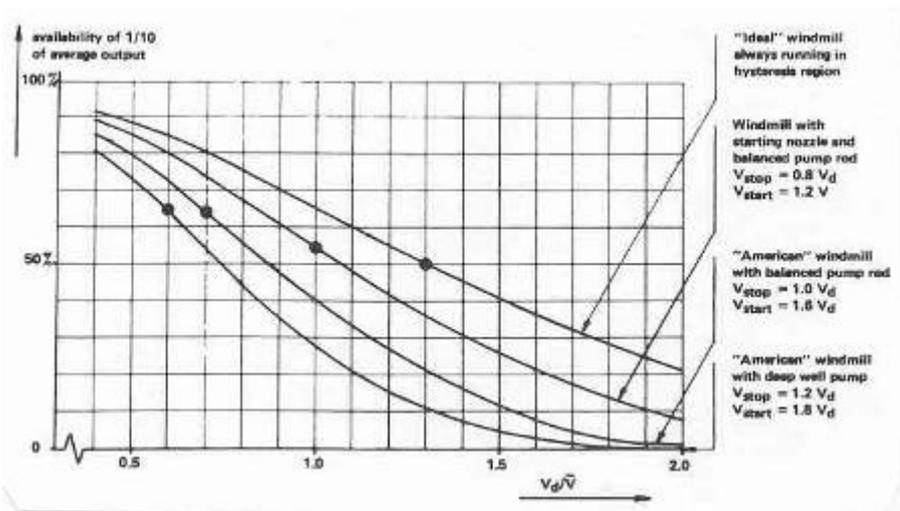
Figure 12   Output availability of better than 10 % of design output in relation to the matching ratio V_d/V_{av} for Weibull factor $k = 2$.

5.2 Choosing the Pump Diameter

Often the range of pump diameters one may choose from is rather limited. For example, CWD pumps exist in four different diameters: 67, 108, 161 and 265 mm for rotor diameters of 2.0, 2.74 and 5.0 mm. Southern Cross has a more extensive pump program, covering 44, 51, 64, 76, 90, 102, 115, 128 and 153 mm for four different rotor sizes: 2.5, 3.0, 3.7, 4.3, 5.2, 6.3 and 7.5 m. The manufacturer generally indicates the head and speed range for which each pump is suitable. CWD instead, gives for each windmill type the design wind speed and the pumping head as a criterion for selection, as shown in Figure 13.
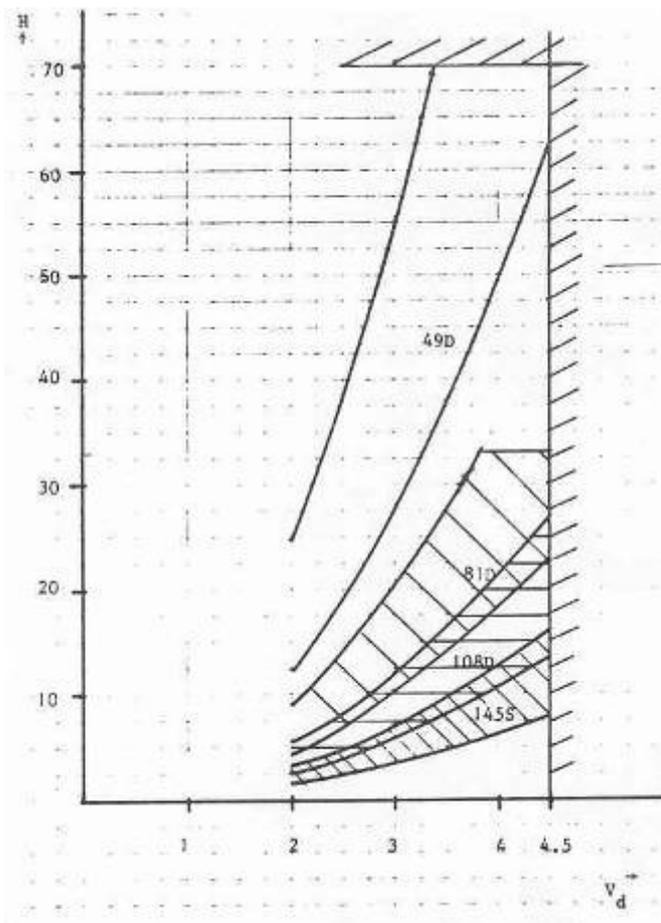
Figure 13   Selection of different sizes CWD pumps according the design wind speed V_d and the pumping head *H*.

5.3 Choosing the Pump Stroke

Once the pump diameter has been chosen, the pump stroke follows directly from *(20)*.

5.4 Controlling the Pump Rod Force

A reliable windpump will have a pump rod capable to cope with the occurring forces under design conditions. For a given head and pump size, these forces vary in congruence with the rotational speed of the pump and the rotor. The total fatigue load on the pump rod depends on the distribution frequency of the pump rod loads, i.e. the load spectrum, and the contribution of the corresponding stresses in the pump rod to the cumulative fatigue.

An adequate pump rod is capable to absorb the cumulative fatigue during the expected

lifetime. Now, if the design speed of the wind pump is increased by attaching a larger pump, the occurring forces at the corresponding wind speeds increase as well. Moreover, the highest occurring loads, i.e. the peak forces at maximum rotational speed, will increase. This increased maximum forces not only contribute to the total fatigue load, but may even exceed the admissable maximum values and the pump rod will break.

The calculation of the total cumulative fatigue load is not dealt with here, but in the following an approximate calculation of the maximum occurring peak load is given. The maximum pump speed is limited by the safety mechanism at ω_{max} corresponding with the rated wind speed of the windpump V_{rated}. At rated wind speed, the wind rotor turns at a tip speed ratio λ close to the maximum tip speed ratio λ_{max}. As for most wind rotors the following relation holds:

$$\lambda_{max} \approx 2\ \lambda_d$$

the maximum pump speed is given by:

$$\omega_{max} \approx 2\lambda_d\ V_{rated}\ /R$$

(34)

This expression is valid in case of an ideal safety system that instantaneously responds to fluctuations in wind speed and direction. In practice, the existing safety systems are less perfect and during short periods, the rotor speed may exceed the theoretical maximum in *(34)* by a factor of 1.5 as seen in Figure 8. So we find as an effective maximum pump speed:

$$\omega_{max,\ eff} \approx 3\lambda_d\ V_{rated}\ /R$$

(35)

Now, we substitute this result into the equation for the pump rod force *(30)* to find the maximum occurring force, and take the overshoot factor k equal to 2:

$$F_{pr} = 2\ F_{st,\ w}\ [1 + \tfrac{1}{2}\ s/g\ (3\lambda_d\ V_{rated}/R)^2\ (A_p/A_{rm})]$$

(36)

This relation should be used to check whether the pump rod is capable to withstand these loads during its expected lifetime.

6 Choice of Materials

The choice of materials for pump components depends on various criteria, such as strength, corrosion resistance, availability, price, ease of manufacturing.
The piston pump and pump rod are perhaps the most critical parts of a windpump. They are subjected to a varying load over a length of years and as such likely to fail because of fatigue. Moreover, they perform their task in an often agressive environment which imposes severe requirements with respect to corrosion resistance. Therefore, materials must be chosen very carefully.

As a basic rule one should avoid combining different metal alloys as this will cause galvanic corrosion. Plastics may be an interesting alternative for lightly loaded parts, especially for shallow well pumping and low water requirements.

6.1 Pump Valves

Valve material may be damaged by erosion and cavitation corrosion. If the velocity of the water in the gap is below 6 m/s, tin bronze will serve. If the water salinity is high, aluminum bronze is a better choice. Stainless steel is superior in quality to bronze but difficult to manufacture and therfore expensive. A good alternative for metal valves is an all-plastic valve set from polyvinylchloride or nylon.

6.2 Cylinder

Most commercial piston pumps are equipped with a pump cylinder made out of brass, bronze, or cast iron with brass lining. The CWD pumps had a stainless steel gas pipe (AISI 316 L, seamless) cylinder, which is a widely available material. A plastic pipe such as PVC can also be used, but will wear quickly.
CWD uses stainless steel for it gives good running properties if combined with a leather sealing. In most countries large stainless steel pipes are commonly available, for example for use in the food processing industry. Bronze as a material may be a good alternative, but large diameter pipes are less strong and more difficult to obtain. Bronze cylinders may also be cast but require machining afterwards, which is less convenient for the larger sizes. Bronze in combination with stainless steel creates a weak galvanic element, but corrosion will be little as both metals are precious.

6.3 Pump Rods

Commercial pump rods are built either from galvanized of stainless steel, or from wood. Galvanized steel cannot be used in an acid environment. The cyclical loading of the pump rod causes fatigue, and the expected lifetime is likely to be reduced by corrosion fatigue. In fact, the pumprod is the most heavily loaded part of a windpump, as it must withstand peak forces upto 5 -10 kN for a large windpump.
Due to its length, a pumprod is generally built up of a number of rod or pipe segments that are welded together. The welds are the weakest spots where the maximum

admissable stresses are lowest. CWD uses galvanized gas pipe, which is cheap, sufficiently strong and readily available.

A galvanized pumprod in combination with one or more bronze pump components creates a very strong galvanic element consisting of a small anodic element (the bottom part of the pump rod) and a large cathodic one (the pump cylinder). A galvanized steel pump rod must be separated from the pump or the water. This can be done with rubber or nylon bushes and rings between the piston and the pump rod. Painting of the pump rod may be an alternative and is more practical. However, the painting layer may not become damaged.

6.4 Rising main

For the rising main, galvanized steel, stainless steel and PVC are possible choices. Galvanized steel is not acid-resistant, while PVC may cause wave phenomena at larger pumping heads due to stretching. For small windpumps of 2.5 m diameter and heads upto 50 m, a PVC can be used without problems.

6.5 Piston Sealing

The most common type of seal used in windpumping is the leather cup. An impregnated and moulded leather cup has proved to be one of the best possible options. Commercial rubber sealings from pneumatic or hydraulic equipment may be suitable as well, but in general give high friction losses.

7 Materials and Corrosion

7.1 Erosion Corrosion

Erosion corrosion is most pronounced on metals protected by a surface layer. An example is aluminum that forms a thin layer of aluminum oxide in contact with the air, which give full protection of the core metal against further corrosion. If exposed to a corrosive flow, such a layer can be continually worn away and the core metal gradually disappears. The sensitivity of a metal to erosion corrosion depends on the surrounding fluid, the flow velocity and the ambient conditions (temperature, salt). In fresh water the effect is relatively small, but under saline conditions (sea water) erosion corrosion can be a serious problem.

Non-alloyed steel and galvanized (zinc) steel have a poor resistance against erosion corrosion, especially in sea water. The spots in a piston pump where erosion corrosion is most likely to occur are the valve gaps where the flow velocities are highest. To reduce the effect of erosion corrosion, as a guideline, the flow velocities in the gap should not

exceed 6 m/s. The average flow velocity in the gap can be reduced by increasing the valve lifting height, however, one should be aware that locally much higher speeds can occur.

7.2 Cavitation Corrosion

Cavitation is called the phenomenon that tiny vapor bubbles (or cavities) are formed in a flow if the pressure locally drops below the vapor pressure of the fluid. This is likely to occur at the back (low pressure) surface of a moving body, such as a screw or a piston. When they collapse they cause a hammering action that deteriorated the surface. This effect is called cavitation corrosion.
Cavitation is most likely in suction pumps at heads of more than 6.5 metre. Especially where cavitation becomes more critical, sharp contours and bends should be avoided as they cause high turbulence and by consequence, additional local pressure drops.
If cavitation corrosion occurs in a piston pump, the cause of cavitation may be eliminated by revision of the pump geometry (rounding edges, modifying valve and seat shape). If this does not work, the pump speed should be reduced. Finally, a material more resistant to cavitation corrosion can be chosen.

7.3 Pitting

Pitting occurs if the effects of corrosion are concentrated at a number of spots rather than attacking the surface uniformly. Small pieces of metal generally will exhibit deep pits, while on large surfaces shallow pits are more common. A piece or iron buried in the soil will corrode forming shallow pits, while a piece of stainless steel in sea water typically exhibits deep pits.
In environments containing ionic chlorine or bromine, all stainless steels tend to corrode forming deep pits. Exposed to sea water, stainless steel develops pits within a few months, the pitting process usually starting at crevices or other areas of stagnant flow.

7.4 Crevices Corrosion

Crevices are called the cavities under or between components, where lack of oxigen may gradually reduce the passivity of the metal surface. The corrosion process is further accelerated by the chemical reactants that become trapped in the crevice and create a highly acid and aggressive environment (pH may become 2 or lower). Crevice corrosion may occur in a cavity in one piece of metal, between two metal bodies, or between a metal and a non-metal body.
Stainless steel is much more sensitive to crevice corrosion than ordinary steel, due to the acidity of the iron, chromium and chlorine corrosion products.

7.5Galvanic Corrosion

Galvanic corrosion occurs if two pieces of metal with a different galvanic potential react with each other due to direct contact or the presence of an electrolyte (sea water). A potential difference exists between two different metals, ot two pieces of the same metal but with different surface properties.

Metals and metal alloys can be ranked according their potential in a galvanic series. The further apart two metals are, the higher the rate of corrosion will be. The metal closest to the anodic end of the series will be the one to corrode. For example, if tin and zinc are in contact, the zinc will corrode; if tin and copper are in contact, the tin will corrode.

The rate of attack is affected by the relative size of the materials, the distance between them and the properties of the electrolyte. A small anodic body combined with a large cathodic area will corrode quickly; conversely, the effect will be less in case of a large anodic area and a small cathodic one. If the distance between the bodies is increased, the effect becomes less.

An electrolyte with good electricity conducting properties facilitates the exchange of electrons between both metals. This explains the fast galvanic corrosion in saline environments and sea water.

Galvanic corrosion can be avoided by a careful choice of materials.

7.6Fatigue Corrosion

When a metal subjected to a periodically varying stress is put into contact with a corrosive medium, the failure limits may reduce dramatically. Under normal conditions, no fatigue occurs if the periodic stresses do not exceed a certain level. This level is called the endurance limit and below this limit, the lifetime of the component is not affected.

Fatigue tests performed in a corrosive environment however, do not exhibit an endurance limit at all: sooner or later the material will always fail. Consequently, one must choose an acceptable lifetime and determine which size and material will do. Tables usually give the number of cycles a material can withstand without failure under given conditions. These data can be used to calculate the expected lifetime.

8Valve Design

The valve design largely determines the flow drag and by consequence the mechanical efficiency of a piston pump and has a strong impact on the dynamic behaviour. Valve design methods only give approximate dimensions, which in practice must be further optimized by practical tests.

A flat valve is relatively easy to produce and often applied in windpumps Figure 14. A valve should have a small contact area with the valve seat to prevent sticking, which lowers the minimum pressure difference needed to open it. A mimimum width of the contact area is 2 to 3 mm in order to guarantee good sealing.

Another type of valve often used is the conical valve as shown in Figure 15. A conical valve however has a less smooth operation and is more difficult to manufacture. The ball valve is found in some of the Southern Cross pumps and the German Lubing (see Figure 16). Other valve types are ring valves, group valves and rubber flappers. As they are not common in windpumps, they are not treated here.

Figure 14   Lay-out of a flat valve design.
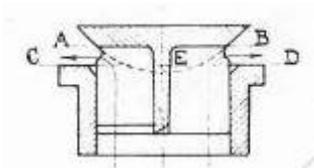

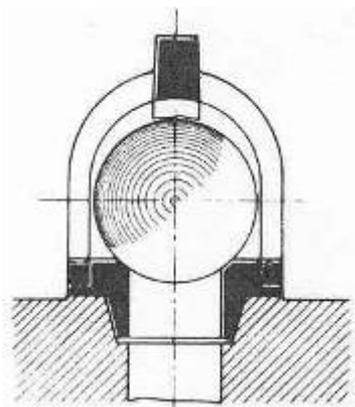Figure 15   Lay-out of a conical valve design.

Figure 16   Lay-out of a ball valve design.

8.1 The Valve Lifting Height

The choice of the valve lifting height is a compromise between the demand for a low

pressure drop over the valve and an acceptable dynamic response (opening and closure delays) at higher pump speeds.

A large valve lifting height results in low flow velocities in the gap, which are necessary to avoid erosion corrosion and cavitation of the valve and the valve seat. Low flow velocities also imply low losses due to flow drag, which improves the mechanical efficiency of the pump.

On the other hand, a large lifting height leads to delayed valve response and by consequence higher peak loads in the pump rod and a more noisy operation. The rotational speed at which a pump starts "hammering" largely depends on the valve design and lifting height and is higher for a low lifting height.

A compromise between both requirements gives practical valve lifting heights ranging from 3 to 12 mm, depending on the pump size.

8.2 Design Procedure Flat Disk Valve

A flat disk valve with central hole is depicted in Figure 17, as well as the relevant diameters and flow areas. To avoid a pressure drop at fully opened valve position, one takes the space between the valve and the cylinder wall equal to the hole area:

$$A_o = A_s$$

(37)

The valve gap space A_g is related to the valve lifting height h according

$$A_g = \pi D_v h$$

(38)

and the space between the valve and the cylinder wall follows from:

$$A_o = \pi/4 \, (D_p{}^2 - D_v{}^2)$$

(39)

The contraction ratio α is defined as the ratio between the gap area and the hole area:

$$\alpha = A_g/A_s$$

(40)

and by applying *(37)* thus:

$$A_o = 1/\alpha \, A_g$$

(41)
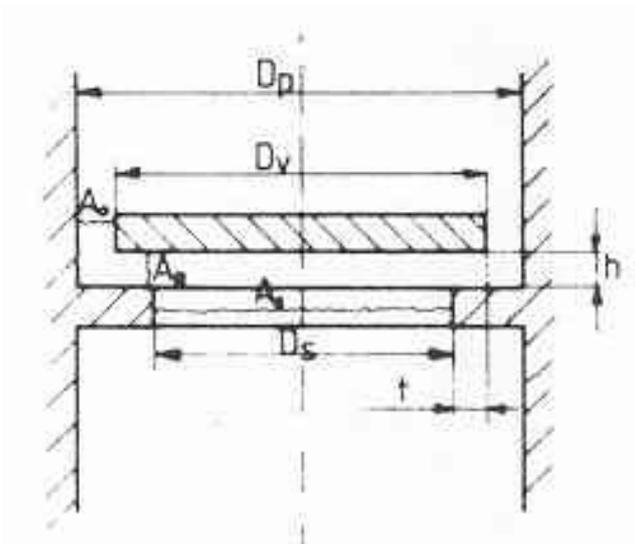
Figure 17   Flat valve model with central hole.

Now by combining *(41)* with the relations *(38)*and *(39)* we obtain:

$$\pi/4\ (D_p{}^2 - D_v{}^2) = 1/\alpha\ \pi D_v\ h$$

(42)

By rewriting we obtain a quadratic expression for the valve diameter as a function of the cylinder diameter, with the valve lifting height and the contraction factor as design parameters:

$$D_v = -2h/\alpha + (4h^2/\alpha + D_p{}^2)^{1/2}$$

(43)

Further, we have to consider an overlap *t* between the valve and the valve seat of 2 to 3 mm to guarantee proper sealing, hence:

$$D_s = D_v - 2t$$

(44)

The final valve geometry is determined from a reasonable choice of the parameters in the latter relations. In order to obtain adequate valve response, the contraction factor should be respected by checking whether:

$$\pi/4\ D_s{}^2 \geq 1/\alpha\ A_g$$

(45)

For other flat valve geometries, such as a ring valve, or more than one seat hole, one can

derive a similar set of equations based on the same assumptions.

9References

Smulders, P. (based on: Trunz, R.), Increasing the Water Output of Windpumps, Small Scale Wind Energy Systems, June 1996, Reric News AIT, Bangkok, Thailand, 1996

Lysen, E.H., Introduction to Wind Energy, CWD Publications, DHV Consultants, Amersfoort, The Netherlands, 1983

Van Meel, J., Smulders, P., Wind Pumping: A Handbook (World Bank Technical Paper No. 101, Industry and Energy Series), The World Bank, ISBN 0-8213-1235-9, Washington DC, U.S.A., 1989

Veldkamp, H.F., On Matching the CWD 5000 Water Pumping Windmill, WM-163, University of Twente, Fac. of Mechanical Engineering, Enschede, The Netherlands, 1989

Diepens, J., Smulders, P., Pump Rod Forces in Wind Pumps, R 1108 D, Technical University of Eindhoven, Fac. of Physics, The Netherlands, 1991

Diepens, J., Design Report CWD 81D Deepwell Pump, Technical University of Eindhoven, Fac. of Physics, The Netherlands, 1991

Southern Cross, The Need for Windmills, Southern Cross Industries, P.O. Box 627, 9300 Bloemfontein, South Africa, 1993

Burton, J.D., Pump Design, Course Module, 1990 and following

10Exercise

10.1Problem

A CWD 2740 windpump has the following characteristics:

rotor diameter:	R	= 2.74	[m]
blade number:	B	= 6	[-]
starting windspeed:	V_{st}	= 2.5	[m/s]
rated windspeed:	V_{rated}	= 7.5	[m/s]
cut-out windpseed:	V_{stop}	= 12	[m/s]
survival windspeed:	V_{surv}	= 50	[m/s]

For this windpump we have two pumps available, a CWD 81D and a CWD 108D type. Both are single-acting piston pumps, the type number indicating the piston diameter in millimetres. The stroke *s* can be adjusted between 0 and 60 mm. The windpump has a balanced pump rod.

From measurements and analysis of the system components, we found that:

volumetric efficiency:	η_{vol}	= 0.9	[-]
mechanical efficiency:	η_{mech}	= 0.8	[-]
design tip speed ratio:	λ_d	= 2.0	[-]
maximum power coefficient	$C_{p,\,max}$	= 0.36	[-]

Furthermore, we have the following constants:

air density	ρ	= 1.2	[kg/m^3]
water density	ρ_w	= 1000	[kg.m^3]
gravitational constant	g	= 9.81	[m/s^2]

> ⇒ 1. Recall the expressions for the design wind speed and the design stroke volume. Sustitute the data given above and derive a simple expression for each of both pumps. Which variables are still unknown?

> ⇒ 2. What are the corresponding design outputs? Compare both expressions.

Measurements on a test site revealed that during gusts, the rotor may speed up until 4 revolutions per second maximally.

> ⇒   3. Give an approximate value for the maximum rotor speed using the theory in this module?

We want to install the windpump above a dug well. The total pumping head H = 20 m. From measurements, we learned that the average wind speed at the site V_{av} = 3.5 m/s.

> ⇒   4. Choose a reasonable design wind speed.

> ⇒   5. Calculate the required pump stroke for both pumps, in order to meet the design point conditions. What is the design output?

The dug well size is wide enough for both pump diameters. The pump cylinders are screwed onto the bottom end of the rising main. The internal diameter of the rising main D_{rm} = 81 mm.

> ⇒   6. Calculate the pump rod forces at the design point and the maximum occuring pump pump rod force, for both pump types.

We decide to use a pump rod made from pieces of ¾" galvanized gas pipe that are welded together. The weakest cross sections of the rod are the welds, where the maximum admissable stresses σ_{max} = 40 N/mm^2. The cross-sectional area A_{rod} = 200 mm^2.

> ⇒   7. Check the strength of this pump rod for both pumps.

After some months of operation, we discover that the water level in the well has dropped by five metres.

> ⇒   8. What has happened to the design wind speed V_d, the rotor speed ω_d and the design output q_d?

> ⇒   9. What can you say about the long term water output and the technical availability? Make us of Figure 11 and Figure 12 if necessary.

$\Rightarrow$ 10. What would you expect for the maximum rotor speed ω_{max}?

$\Rightarrow$ 11. What has happened to the pump rod force at the design point, and at maximum speed? Discuss the effect on fatigue of this component?

10.2 Answer

A CWD 2740 windpump has the following characteristics:

rotor diameter:	R	= 2.74	[m]
blade number:	B	= 6	[-]
starting windspeed:	V_{st}	= 2.5	[m/s]
rated windspeed:	V_{rated}	= 7.5	[m/s]
cut-out windpseed:	V_{stop}	= 12	[m/s]
survival windspeed:	V_{surv}	= 50	[m/s]

For this windpump we have two pumps available, a CWD 81D and a CWD 108D type. Both are single-acting piston pumps, the type number indicating the piston diameter in millimetres. The stroke *s* can be adjusted between 0 and 60 mm. The windpump has a balanced pump rod.

From measurements and analysis of the system components, we found that:

volumetric efficiency:	η_{vol}	= 0.9	[-]
mechanical efficiency:	η_{mech}	= 0.8	[-]
design tip speed ratio:	λ_d	= 2.0	[-]
maximum power coefficient	$C_{p,\,max}$	= 0.36	[-]

Furthermore, we have the following constants:

air density	ρ	= 1.2	[kg/m^3]
water density	ρ_w	= 1000	[kg.m^3]
gravitational constant	g	= 9.81	[m/s^2]

> ⇒ 1. Recall the expressions for the design wind speed and the design stroke volume. Sustitute the data given above and derive a simple expression for each of both pumps. Which variables are still unknown?

Formula *(19)* gives:

$$V_d = \sqrt{(\eta_{vol}\, s\, A_p\, \lambda_d\, \rho_w\, g\, H) / ((C_p\, \eta)_{max}\, \rho\, \pi^2\, R^3)}$$

By subsitution of all given parameters we find for the design wind speed:

$$V_d = \sqrt{(0.9\, s\, A_p * 2.0 * 1000 * 9.81 * H) / (0.36 * 0.8 * 1.2 * \pi^2 * 1.37^3)}$$

$$= 44.87 \sqrt{(s\, A_p\, H)}$$

Hence the design stroke volume is equal to:

$$s\,A_p = 1/(2013) * (V_d^2/H)$$

The piston area is different for each of both pump. We find:

type	D_p [m]	A_p [m^2]	V_d [m/s]	s [mm]
CWD 81D	0.081	0.00515	$3.22\sqrt{(s\,H)}$	$96.5\,(V_d^2/H)$
CWD 108D	0.108	0.00916	$4.29\sqrt{(s\,H)}$	$54.2\,(V_d^2/H)$

The pumpstroke s, pumping head H and design wind speed V_d are still unknown.

⇒ 2. What are the corresponding design outputs? Compare both expressions.

The design output q_d follows from equation *(18)*:

$$q_d = \eta_{vol}\,s\,A_p\,\lambda_d\,V_d/(2\pi R)$$

We subsitute and find:

$$q_d = 0.9\,s\,A_p * 2.0 * V_d/(2\pi * 1.37)$$

$$= 0.209\,s\,A_p\,V_d$$

$$= 0.209/2013\,(V_d^3/H) \qquad [m^3/s]$$

$$= 0.104\,(V_d^3/H) \quad [l/s]$$

This expression is independent from the chosen pump type (of equal efficiency).

Measurements on a test site revealed that during gusts, the rotor may speed up until 4 revolutions per second maximally.

⇒ 3. Give an approximate value for the maximum rotor speed using the theory in this module?

We use the relation *(34)*:

$$\omega_{max,\,eff} \approx 2\lambda_d\,V_{rated}/R$$

and find:

$$\omega_{max,\,eff} \approx 2 * 2.0 * 7.5/1.37$$

$$\omega_{max,\,eff} \approx 21.9\ [rad/s]$$

This is equivalent to 21.9/6.28 = 3.5 revolutions per second. The maximum speed of 4 rev/s found in practice is slightly higher (but not as much as a factor 1.5 as described in the module).

We want to install the windpump above a dug well. The total pumping head $H = 20$ m. From measurements, we learned that the average wind speed at the site $V_{av} = 3.5$ m/s.

$\Rightarrow$ 4. Choose a reasonable design wind speed.

As a first choice, take V_d equal to the annual mean wind speed:

$$V_d = V_{av} = 3.5 \; [m/s]$$

The corresponding design pump speed $\omega_d = 5.1 \; rad/s$.

$\Rightarrow$ 5. Calculate the required pump stroke for both pumps, in order to meet the design point conditions. What is the design output?

We can now complete the last column in the table from question 1 by substituting $V_d = 3.5$ and the head $H = 20$ m:

type	D_p [m]	A_p [m^2]	s [mm]
CWD 81D	0.081	0.00515	59.1
CWD 108D	0.108	0.00916	33.2

The stroke volume $s \, A_p = 0.304$ litre for both pumps.

The design output q_d is:

$$q_d = 0.104 \; (3.5^3/20) = 0.22 \, [l/s]$$

The dug well size is wide enough for both pump diameters. The pump cylinders are screwed onto the bottom end of the rising main. The internal diameter of the rising main $D_{rm} = 81$ mm.

$\Rightarrow$ 6. Calculate the pump rod forces at the design point and the maximum occuring pump pump rod force, for both pump types.

We recall the expression for the pump rod forces *(30)* or *(31)*:

$$F_{pr} = k \, F_{st, \, w} \, [\; 1 \; + \; \tfrac{1}{2} \, s \; \omega^2/g \; (A_p/A_{rm}) \;]$$

$$F_{pr} = k \, F_{st, w} \, [\, 1 + c_a \, (A_p/A_{rm}) \,]$$

At design point, we calculate the rotational speed from:

$$\omega_d = \lambda V_d/R$$

$$= 2.0 * 3.5/1.37 = 5.1 \; rad/s \; (or \; 0.81 \; rev/s).$$

The maximum rotational speed of 4 rev/s is, hence $\omega_{max} = 25.1$ rad/s.

The static force is found from *(1)*:

$$F_{st, w} = \rho_w \, g \, H \, A_p$$

Bearing in mind that the ratio A_p/A_{rm} is different for both pumps, we calculate the pump forces in the following table (taking the overshoot factor equal to 2):

type	A_p [m^2]	$F_{st, w}$ [N]	A_p/A_{rm}	s [mm]	c_a (ω_d)	c_a (ω_{max})	$F_{pr, d}$ [N]	$F_{pr, max}$ [N]
CWD 81D	0.00515	1010	1.0	59.1	0.0785	1.90	2179	5858
CWD 108D	0.00916	1797	1.33	33.2	0.0440	1.07	3804	8708

We decide to use a pump rod made from pieces of ¾" galvanized gas pipe that are welded together. The weakest cross sections of the rod are the welds, where the maximum admissable stresses $\sigma_{max} = 40$ N/mm^2. The cross-sectional area $A_{rod} = 200$ mm^2.

> ⇒ 7. Check the strength of this pump rod for both pumps.

For the CWD 81D we calculate the maximum occurring stress:

$$\sigma_{max} = 5858 / 200 = 29 \; [N/mm^2]$$

For the CWD 108D we find:

$$\sigma_{max} = 8708 / 200 = 43 \; [N/mm^2]$$

In case of the CWD 108D, the maximum occurring stresses in the pump rod exceed the maximum admissable value.

After some months of operation, we discover that the water level in the well has dropped by five metres.

⇒   8. What has happened to the design wind speed V_d, the rotor speed ω_d and the design output q_d?

The design wind speed has changed, which is concluded from the expression found in question 1:

$$V_d = 44.85 \sqrt{(s\, A_p\, H\,)}$$

Due to this change in design speed, the design pump speed ω_d has also changed and therefore the design output (by definition, at the design point, the rotor operates at $\lambda = \lambda d = 2.0$):

$$q_d = 0.209\; sA_p\, V_d$$

we now calculate for the new head $H = 25\ m$:

$$V_d = 44.85 \sqrt{(0.00304 * 25)} = 3.91\ [m/s]$$

The increased design output becomes:

$$q_d = 0.209 * 0.304 * 3.91 = 0.25 \quad [l/s]$$

$$(\text{or: } q_d = 0.104\ (3.91^3/25) = 0.25 \quad [l/s]$$

So the design output has increased by about 10%.

⇒   9. What can you say about the long term water output and the technical availability? Make us of Figure 11 and Figure 12 if necessary.

The design wind speed V_d has shifted from 3.5 to 3.91 m/s. The average wind speed V_{av} is still 3.5 m/s, hence the matching ratio V_d/V_{av} has changed from 1 to 1.12. Now we can read in the figures, -following the curve for the "American" windmill with balanced pumprod- that the dimensionless output has increased from 0.6 to 0.52 and the availability from 40% to 32%.
So the output has increased by 15%, at the expense of a reduction of the availability of 20%.

⇒   10. What would you expect for the maximum rotor speed ω_{max}?

As V_{rated} is controlled by the safety mechanism it should not change. Since at high speeds the rotor operates approximately at maximum tip speed ratio, the ω_{max} should remain the same.

⇒   11. What has happened to the pump rod force at the design point, and at maximum speed? Discuss the effect on fatigue of this component?

Since ω_d has increased, the pump rod loads at the design point have increased as well.

type	A_p [m^2]	$F_{st,\,w}$ [N]	A_p/A_{rm}	s [mm]	ω_d [rad/s]	$c_a\,(\omega_d)$	$F_{pr,\,d}$ [N]
CWD 81D	0.00515	1010	1.0	59.1	5.71	0.0982	2218
CWD 108D	0.00916	1797	1.33	33.2	5.71	0.0552	3858

The maximum occuring pump rod forces have not changed, but the loads spectrum has shifted towards higher values. This increases the accumulated fatigue in the pump rod, reducing its life time. But in this case the acceleration component at design conditions is less than 10% of the static force ($c_a < 0.1$), so we should not experience any problems.

---o---